DETERMINATION OF SIPHON OPERATING REGIMES

A. I. Kuprin and V. N. Gotsulenko UDC 532.14(088.8)

Necessary and sufficient conditions are found for continuous and periodic liquid motion in systems containing siphons.

Siphons can be used to move liquids from vessels in a continuous operation regime. Moreover, our studies have shown that siphons operating continuously can be used for precise measurements of volumetric flow rates in open hydrosystems, since the flow rate is proportional to Δh (Fig. 1). In the range of low flow rates Q the continuous mode of siphon operation is disrupted. Discontinuous self-oscillations are then excited in the hydrosystem [1]. The periodic regime of siphon operation is the basis of automatic liquid dosing devices [2-6], which have found use in the chemical, pharmaceutical, food, and other industries.

At the same time it should be noted that over the entire range of flow rates Q where the periodic regime occurs the siphon can also operate in the continuous regime if the region is entered from the side of higher flow rates.

The theoretical treatment offered below and the form of siphon characteristics obtained experimentally permit determination of the features of siphon operation and establishment of the cause of periodic operation.

To determine the conditions for excitation of self-oscillations in the hydrosystem including a siphon (Fig. 1) we write the equations of nonsteady state liquid flow for such a system.

The hydraulic oscillatory circuit consists of the acoustic compliance of the liquid column in the measurement volume C_a, and the acoustic mass (inertia) L_a of the siphon tube.

The change in liquid mass dm in the measurement volume over time dt is proportional to the difference between the flow rates

$$dm = \rho (Q_{in} - Q)\, dt.$$

Considering that the liquid mass in the measurement volume $m = \rho s_e(h + h_0)$, while the pressure at the siphon inlet $p = \rho gh$, we obtain the equation for the change in mass in the form

$$C_a \frac{dp}{dt} = Q_{in} - Q, \tag{1}$$

where $C_a = s_e/\rho g$, h_0 = const (Fig. 1).

The rate of flow of liquid entering the measurement volume Q_{in} depends on the pressure difference $p_e - p$ and the hydraulic resistance $R(Q_{in})$ and can be determined from the equation

$$p_e - p = R(Q_{in}). \tag{2}$$

According to [7] the equation for change in momentum of the liquid through the siphon with consideration of its hydraulic resistance h(Q) can be written as:

$$L_a \frac{dQ}{dt} = p - h(Q), \tag{3}$$

where $L_a = \rho l/s$, l is the length of the siphon tube.

Eliminating the variable Q_{in} from Eqs. (1)-(3), we obtain a system of two first order nonlinear differential equations describing nonsteady state siphon operation:

$$L_a \frac{dQ}{dt} = p - h(Q); \qquad C_a \frac{dp}{dt} = f_1(p) - Q. \tag{4}$$

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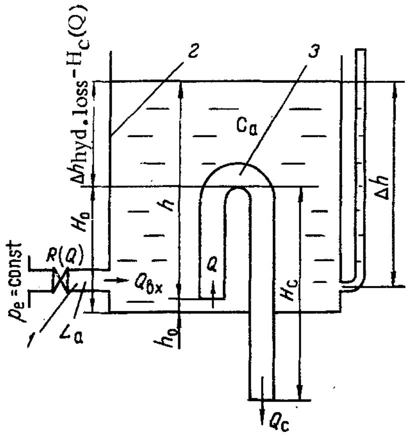

Fig. 1

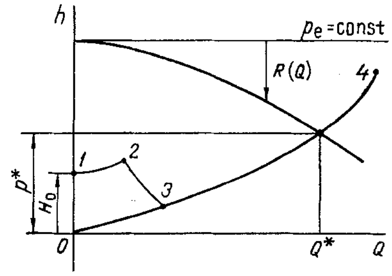

Fig. 2

Fig. 1. Diagram of hydrosystem containing siphon: 1) supply pipe; 2) measurement vessel; 3) siphon.

Fig. 2. Siphon hydraulic characteristic and conditions for simultaneous operation in hydrosystem.

The function $f_1(p)$ is a convolution of the function $p = f(Q_{in})$, where $f(Q_{in}) = p_e - R(Q_{in})$ and p_e = const.

Let Q^* and p^* be values corresponding to an arbitrarily chosen siphon operating regime which is determined by solution of the system of equations $p - h(Q) = 0$; $p_e - R(Q) - P = 0$ or the equation of simultaneous operation of the hydraulic energy source at p_e = const and the hydrosystem with siphon (Fig. 2):

$$p_e - R(Q) = h(Q). \tag{5}$$

We translate the origin of the coordinate system to the singular point Q^*, p^*, setting $Q = Q^* + x$, $p = p^* + y$; we write the nonlinear functions $h(Q)$ and $f_1(p)$ in the form of Taylor series about the operating point of the regime.

In the new coordinates system (4) can be written as:

$$\begin{aligned} L_a \frac{dx}{dt} &= y - h'x - o(x^2), \\ C_a \frac{dy}{dt} &= f_1' y + o(y^2) - x, \end{aligned} \tag{6}$$

where

$$h' = \left.\frac{dh}{dQ}\right|_{Q^*}, \quad f' = \left.\frac{df}{dQ}\right|_{Q^*}, \text{ while } f_1' = \frac{1}{f'}.$$

The equation of the first approximation to Eq. (6) will have the form

$$L_a \frac{dx}{dt} = y - h'x; \quad C_a \frac{dy}{dt} = f_1' y - x. \tag{7}$$

The characteristic polynomial of system (7) is:

$$C_a L_a \lambda^2 + (C_a h' - L_a f_1')\lambda + (1 - f_1' h') = 0. \tag{8}$$

Following [8] we note that the value of the derivative $R|_{Q^*} = 1/k$, then $k = -1/f_1'$, since $df/dQ = dR/dQ$.

The static stability condition is [8]

$$k + h' \geqslant 0, \tag{9}$$

with dynamic stability condition

$$\frac{L_a}{kC_a} + h' \geqslant 0. \tag{10}$$

According to inequalities (9), (10) the necessary condition for excitement of self-oscillations is the appearance of a descending branch in the siphon characteristic h(Q), where the value of the derivative $dh/dQ < 0$.

Figure 2 shows the dependence of hydraulic resistance on flow rate Q of the liquid entering the siphon: $h(Q) = H_0 + \Delta h_{hyd.loss} - H_c(Q)$. The characteristic branch 1-2 is the dependence of hydraulic resistance on flow rate with the siphon not operating (the siphon head level is not reached yet). The branch 2-3 defines the reduction in hydraulic resistance when the siphon begins to operate and its head increases as the section completely filled by liquid moves down the descending siphon tube. The dependence of hydraulic resistance on flow rate for continuous siphon operation is defined by ascending branch 0-3-4. When the operating point is located on the descending branch of the characteristic 2-3, when the value of the derivative $h' < 0$, either one or both of the stability conditions following from inequalities (9), (10) is not satisfied. Therefore the siphon operating mode in this region of the characteristic is periodic with a limiting cycle 0-1-2-3-0 (Fig. 2).

Discontinuous self-oscillations are then excited in the hydrosystem [1], in which the liquid flow rate through the siphon changes discontinuously. If we connect an acoustic capacitance to the ascending or descending siphon tube, experiments show that the character of the self-oscillations remains extremely nonharmonic, but the liquid flow may become continuous. Thus in the given case we have established the possibility of transition from discontinuous self-oscillations to conventional relaxation phenomena. It should be noted that the presence of only a single energy storage device (acoustic capacitance) is not an attribute of a relaxation state as defined in [8]. The cause of the relaxation nature of a hydrosystem including a siphon is that filling of measurement volume 2 (Fig. 1) occurs slowly while it is emptied rapidly, which is the main determinant of the character of the motion.

A continuous sophon operating regime in the unstable region of the characteristic can be insured by constancy of the pressure in the measurement volume, where $C_a = \infty$, i.e., the measurement volume must have a large section S_e, or the condition $Q_{in} \geqslant Q_s$ must be satisfied.

NOTATION

Q_{in}, Q, Q_s, volumetric flow rate of liquid entering measurement volume, and leaving siphon; p, pressure at siphon inlet; x, y, flow rate and pressure deviations; L_a, C_a, acoustic mass and compliance; t, time; s_e and S, cross sectional area of measurement volume and siphon tube; $R(Q)$, $h(Q)$, hydraulic characteristics of supply line to oscillatory circuit and siphon; ρ, liquid density.

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